



The Effective Integration of Students' Beliefs, Goals and Learning Strategies for Enhancing Problem Solving Skills at Higher Secondary Level

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Abstract

In the modern educational settings, the relationship between students' beliefs and the learning strategies to solve a problem has become important. Particularly, epistemological may greatly influence problem-solving skills through shaping students' goal orientation beliefs and learning strategies. Overall, epistemological beliefs, goal orientation beliefs, SRL had prominent effects for teaching mathematics and problem solving. Therefore, for the current study, combining these concepts were considered for further investigation. To analyze the relationship among different constructs, a comprehensive model was proposed. For the specified objectives, five diverse questionnaires were used. The relationship among different constructs, a correlational design method was used for the 430 participants. Target population was the students, studying in higher-secondary level and intermediate colleges in a province "Khyber Pakhtoon Khawa (KPK)" Pakistan. The results had shown strong correlation among beliefs, goal orientations, SRL and problem solving. The results also specified the prominent mediation role of mastery and performance goals, and elaboration. Whereas the role of avoidance and critical thinking was insignificant. The findings revealed how these factors could enhance students' ability to solve problems in real-world situations. The study also emphasized the negative effects of parents' unrealistic expectations and superficial memorization that is solely concerned with obtaining good grades. Implications have been proposed to encourage more efficient and significant learning in accordance with these findings.

Keywords: epistemological and motivational beliefs; problem solving; self-regulating learning; structural equation modelling.

1 Introduction

Problem solving is vital in teaching mathematics as it engages students' critical thinking skills and promotes creativity [1]. For that reason, problem solving [2] mainly focus on unravelling real world problems [8]. However, solving complex tasks are mostly non-routine and considered hard jobs [19]. These require the concrete knowledge and tools of modelling, analyzing and optimizing real world phenomena [51]. In addition, the modelling and optimizing activities, role of differential equations (DEs) is considered most prominent [14]. Consequently, differential equations courses at secondary schools have become a crucial component of the curriculum [35]. For pre-university students to be prepared for the rigorous standards of university-level education in math, physics, and engineering, a strong emphasis on DE problem-solving skills and a solid base in subject matter are essential. Students can be better prepared for university-level courses and achieve academic and professional success in disciplines that demand an in-depth comprehension of DEs by having a strong pre-university expertise in mathematics, physics, and engineering [8]. It may therefore enhance the chance of academic and professional success in disciplines that demand an adequate understanding of DEs [49]. The post-secondary level has emerged as an essential part for problem solving competence at higher level.

Five main approaches have been noticed in the literature for teaching and learning DEs: algebraic, graphical, numerical, technology based and inquiry-oriented [8]. The algebraic approach focuses on using symbolic manipulation and mathematical reasoning to solve DE problems. The graphical approach involves visualizing and interpreting DE solutions using graphs and diagrams [49]. The numerical approach employs numerical methods and computer software to approximate and solve DEs. The technological approach utilizes computer-based tools and software to facilitate the learning and teaching of DEs. The inquiry-oriented approach emphasizes exploration, experimentation, and discovery to foster deeper understanding and critical thinking about DEs.

Research has shown that at higher levels of education, a technological-based approach for teaching and learning DEs has yielded better results compared to other approaches. The use of computer-based tools and software allows for interactive and dynamic visualization of DE solutions, making it easier for students to understand and engage with the subject matter [46]. Understanding how students engage with digital tools and representations is essential in promoting student learning and ensuring that technology is used effectively in the teaching and learning of DEs [30]. Over all algebraic, graphical, and numerical based approaches were found dominant at lower level. It was also revealed that students' have better skills in solving algebraic tasks. In numerical-based task solving, although students have demonstrated better outputs during understanding and planning phases. However, lesser students were able to give the accurate answer with proper units. Similarly, a few were able to solve DEs graphically. Previous literature has also revealed that the factors such as the limited availability of technological tools and the need to prioritize basic mathematical skills in early education, use of the algebraic approach is common, particularly in developing countries.

Besides to aforementioned domains, context familiarity also impacted students' problem-solving skills [11]. Compared to biologically based problems, students remained successful when solving physics-related tasks. The percentage of physics and mathematics taught proportion was found to be higher than the percentage of biology, resulting a more familiar context [18]. Interestingly, students had shown best results while solving compound interest-based tasks. This might be the result of awareness about topics. Usually, "interest" or "compound interest" is applied in connection with banking, and finance. Generally, social media, news networks, and television use the terminology of compound interest which had helped the students while solving the task. As a

result, the students' performance had improved.

Additionally, solving non-routine mathematical problems has been taken into account along with context familiarity [50]. These tasks which are usually new, unforeseen, and unscheduled, necessitate that students use extra dedication and learning techniques [41]. These types of tasks can particularly provide a more realistic way to solve complex real-world problems, preparing students for future applications of DEs in their careers and daily lives. Non-routine tasks are frequently avoided by learners because of their complexity [44]. Their ability to comprehend the concepts in higher education is severely hampered by this avoidance, as it also makes it difficult for them to apply whatever they have learned to real-world situations. Assessments involving only routine-based problem-solving cannot provide solid ground for the transition of secondary to tertiary levels [36]. Beyond context familiarity and problem-solving skills, students' epistemological beliefs regarding mathematics have been demonstrated that they had a substantial impact on their ability to solve DES problems [47]. These beliefs provide an association between an individual's views about knowledge, knowing and how these epistemological premises impact those thinking and reasoning cognitive processes [42].

For the current study, epistemological beliefs were considered for solving a task. Like epistemological beliefs, usefulness can also increase learners' motivation and problem solving skills [48]. Several studies have further linked math achievement and SRL strategies [28]. Self-regulation is the cyclical process by which a student sets a task, retains monitoring his performance, and evaluates the result. In general, self-regulation is the cyclical process by which a student sets a task, retains monitoring his performance, and evaluates the result. Stockton [16] had proven the significance of self-regulation in problem-solving, indicating that scholars who were skilled at monitoring and directing their learning processes score higher on problems-solving tasks. Kizilgunes et al. [30] found an association between epistemological beliefs, learning strategies and math achievements. The following sections will provide further detail.

1.1 Epistemological beliefs and goal orientation

Personal epistemology, which is defined as beliefs about knowledge and knowing. Goals, values [23] and reasoning processes [4] are all aspects of epistemic cognition, encompassing personal epistemology. These beliefs had been shown to influence an array of aspects of academic motivation, learning, and achievement. One way that epistemic beliefs impact learning is the standards and objectives that students set, have an impact on their reflexive metacognitive monitoring of their cognitive processes that correspond to various learning tasks [32]. Students who hold sophisticated epistemological views—such as the idea that knowledge is a dynamic and ever-evolving entity—are more likely to adopt mastery goal orientations, which foster a passion for learning and resilience when confronted with of adverse situations. Individuals with naive epistemological concepts, on the other hand, might be inclined toward performance orientations and value grades over understanding. Epistemological beliefs have a significant impact on how students perceive and understand content [13]. The advantages of complex epistemic beliefs include better learning outcomes, more capacity for reflective thought, and higher critical thinking abilities.

1.2 Goal orientation and SRL strategies

Goal orientation and SRL strategies are interconnected concepts which interact collectively to affect students' academic achievement, motivation, and engagement. Mastery oriented students

are intrinsically motivated to acquire an extensive understanding of the subject matter. Deep learning strategies like elaboration, self-monitoring, and reflection—all essential elements of SRL—are more likely to be employed by them. This frame of mind encourages proactive planning, self-control, and perseverance in challenging tasks [34]. Performance goals-oriented students are more likely to concentrate on displaying their abilities, which frequently results in surface learning strategies like rote memorizing. However, if their focus is on surpassing their classmates, some students with performance-approach goals may still use effective SRL strategies [37]. Performance-avoidance goal orientation was a negative predictor of interest [38], self-efficacy for self-regulated learning and writing self-concept and intrinsic motivation [7]. Motivational beliefs significantly predicted SRL strategies, with intrinsic goal orientation and task value being the most significant predictors of nine SRL strategy sub-factors [27].

1.3 Problem solving with epistemological beliefs, goals orientation and SRL strategies

Epistemological beliefs impact goal orientation; students who see knowledge as complicated and constantly evolving are more likely to pursue mastery goals. Goal orientation and epistemological beliefs determine the selection and use of learning strategies; deeper strategies are encouraged by more complex beliefs and mastering goals-oriented students. Effective learning techniques improve problem-solving abilities, particularly when they are aligned with mastery goals and adaptable epistemological beliefs [3]. Overall, it was concluded that epistemological beliefs, goal orientation beliefs, SRL had notable effects for teaching and learning mathematics. Therefore, for the current study, these aspects were considered for further investigation. Combining these concepts, educators and curriculum designers can further guide the development of effective teaching strategies that address students' beliefs about mathematics, their understanding of its usefulness, their ability to engage in self-regulated learning, and their goal orientation towards mathematical problem solving.

2 Conceptual Frame Work

Numerous studies are evident that epistemological beliefs strongly influence an array of aspects of academic motivation, learning, and achievement. Based on the studies cited, a strong association is found between self-regulated strategies and mathematics achievements. In this regard, Stockton [49] revealed that students perform better on math solving tasks when they properly monitor and control their own learning processes. In addition to these beliefs and SRL, it was also observed that goal orientation beliefs can also influence an individual's self-regulatory behavior, as they affect the implementation of self-regulatory knowledge and skills [28]. Kizilgunes et al. [30] had also associated epistemological beliefs, goal orientation beliefs, learning approaches with academic achievements. Goal orientations were significantly related to mathematics problem-solving performance. Specifically, students who were motivated by mastery goals (i.e., improving their own skills and knowledge) performed better than those who were motivated by performance goals (i.e., seeking external validation and approval) [48]. However, avoidance goal-orientated students develop restrictive mindsets such as fear of being rejected or failing as a result mostly students give up in unfamiliar and challenging circumstances which has little impact on their mathematical achievement. These findings suggested that performance-oriented students tend to focus on achieving high grades or outperforming others, while mastery-oriented students focus on learning and understanding the material. Avoidance-oriented students, on the other hand, tend to avoid failure and negative outcomes rather than seeking success [15]. Mastery and performance goals have been found to be predictors of SRL strategies, with mastery goals being

the most effective in generating positive outcomes.

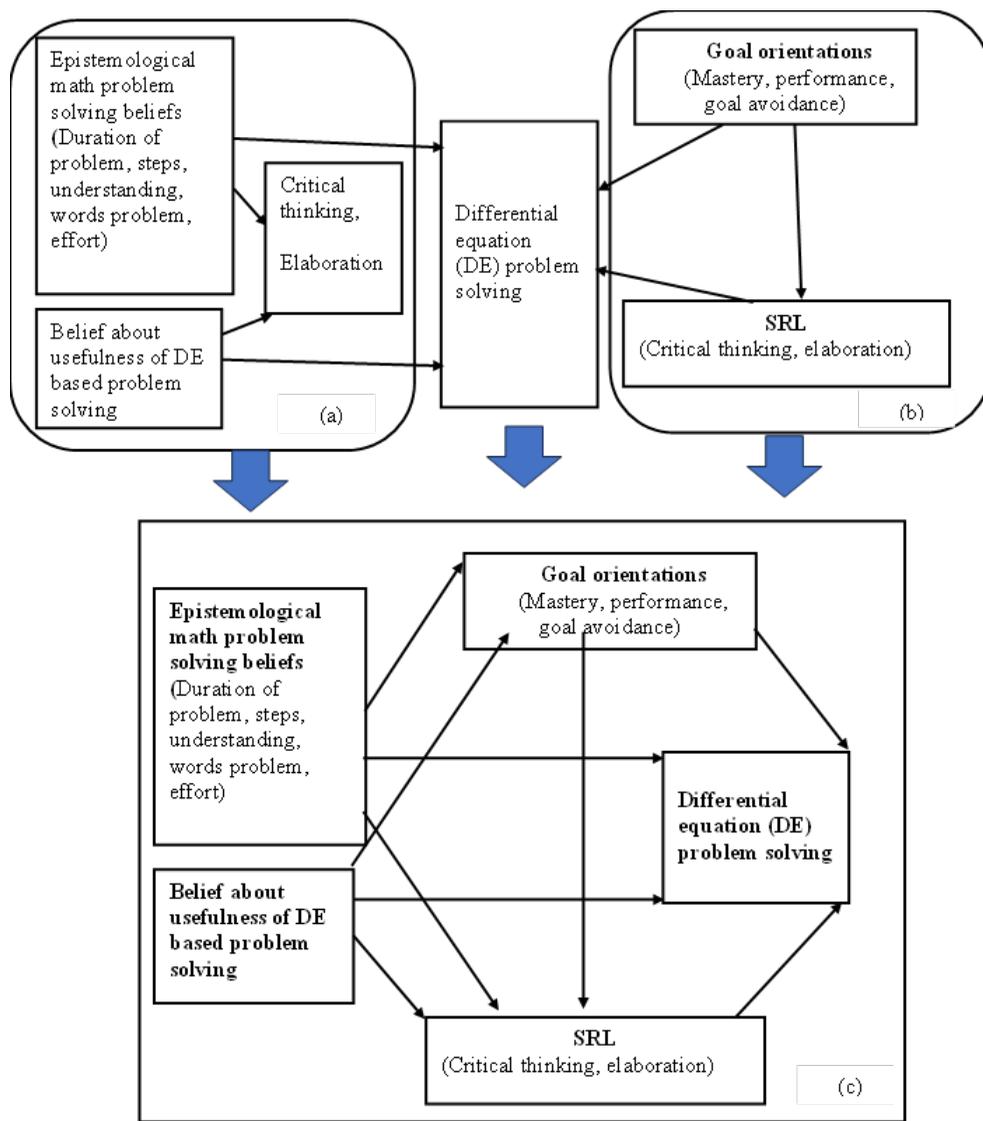


Figure 1: Proposed framework with selected constructs.

Overall, epistemological beliefs, goal orientation beliefs, and SRL had prominent effects for teaching mathematics and problem solving. Therefore, for the current study, combining these concepts were considered for further investigation. Figure 1 illustrates the conceptual framework for the current study. In the previous work, role of SRL between goal orientations and problem solving was investigated [23]. Also, role of goal orientations between beliefs and problem solving was investigated [5]. The results were highly encouraging [4]. In current study, a complete structural equation model has been developed by combining model (a) and model (b) to explain complete relationship of goal orientations and SRL amongst beliefs and problem solving. Figure 1 (a, b & c) illustrates the complete avenue of the current study. By knowing how various factors both directly and indirectly affect student learning, educators may devise higher-quality teaching strategies to help students succeed in differential equations and other math courses. The proposed

conceptual framework highlights the interrelationship between these constructs.

3 Research Objectives

The research aimed to explore the factors influencing DE problem solving at post-secondary level, specifically, focusing on identifying direct and mediating effects:

1. To determine the direct effect of four selected factors on students' DE problem solving.
2. To investigate whether goal orientation (i.e., the motivation behind learning) and students' self-regulated learning strategies acts as mediators between students' beliefs and their problem-solving abilities.
3. To analyze whether goal orientation and students' self-regulated learning strategies act as mediators between students' perceptions of the usefulness of mathematics and their problem-solving abilities.
4. To analyze whether students' self-regulated learning strategies act as mediators between students' goal orientation and their problem-solving abilities (model inside pattern).

4 Materials and Methods

4.1 Participants

To analyze the specified objectives, a proposed model Figure 1 was used. The relationship among different constructs, a correlational design method was used to identify their effect on problem-solving. For current study, target population was the students, studying in higher-secondary level and intermediate colleges in a province "Khyber Pakhtoon Khawa (KPK)" Pakistan. After piloting the instruments, research instruments were distributed among 430 participants. The selected sample size was deemed appropriate for representing the population, allowing the generalization of the findings. Its diversity, achieved by drawing from both government and private institutes, yielded more inclusive understandings into the aspects influencing mathematics problem-solving in Pakistan.

4.2 Research instruments: Questionnaires and non-routine tasks

Four questionnaires and an assessment tool containing five problems were used to examine the effects of selected factors. All items were written in both Urdu and English languages for easy and better understandings. To confer the credibility of instrument, mathematicians and educators who were teaching at the university and post-secondary school levels in Pakistan were consulted to evaluate the content, face validity, clarity, and translation of the instruments used in the study. Additionally, the researchers also sought feedback on the suitability of the non-routine DE tasks used in the study. To ensure the accuracy of the translation of the instruments into Urdu, tools were distributed among participants and researchers who were proficient in both Urdu and English. This step was important to ensure that the translation is accurate and that the instruments can be

understood by the participants in the study. Basit and Rahman [2] emphasized that this technique could resolve the linguistic issue.

These instruments were further studied through a cross-sectional survey that is well suited to assess interrelationships among various factors within a population. The accurate data required for hypothesis testing also depend on the selection of an adequate sample size representing the population truly. By employing cluster sampling technique, the study ensured that any population bias was equally distributed among the selected individuals.

This study employed the modified Indiana Mathematics Beliefs Scale (IMBS) across five dimensions: time duration, steps, word problems, understanding, and effort. Each dimension comprised six items (three positive and three negative) rated on a 5-point scale with the subscales demonstrating the reliability of 0.73. Usefulness was measured using the modified Deemer [13] which had a Cronbach's alpha of 0.86.

The second instrument used to measure goal orientations was the Patterns of Adaptive Learning. This instrument consisted of three dimensions: mastery, performance, and avoidance goals. Mastery and avoidance goals were assessed using six items each, while performance goals were assessed using five items. All of these items were ranked from 1 (strongly disagree) to 5 (strongly agree). The reported reliability for the three subscales were 0.86 for mastery goals, 0.86 for performance goals, and 0.75 for avoidance goals. Self-regulated learning strategies were assessed using two dimensions from the Norwegian adaption of the Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich, 1991) elaboration (six items) and critical thinking (five items). Both scales are rated on 1 – 7-point Likert scale (1 = Not at all true to 7= Very true). Reliability values for these subscales were previously reported as 0.75 and 0.80, respectively [37].

In present study, word problems were also developed to assess the students' ability to solve differential equations (DEs) in real-world contexts. Algebraic methods were needed to solve these first three tasks. On the contrary, task 4 was about health and disease. Students' have to utilize graphical based knowledge and skills for solving task 4. Similarly, task 5 was a puzzle problem involving baking of food items (a specific case of Newton's law of heating). Students were given additional data for both particular and general solutions. It was observed how students handle these conditions. A scoring rubric of Charles *et al.* [7] was adapted to analyze the scores of the solved tasks.

4.3 Procedures

To check the reliability of the research instruments, mathematicians and educators were consulted to evaluate the content, face validity, and clarity. In the next step, instruments were piloted and finalized based on the feedback received from 250 students. Analyzing the pilot study results, it was confirmed that all the scales used in the study were reliable, as the values of internal consistency were exceeding 0.7. This was a positive finding, as it indicated that the instruments are likely to measure the constructs they are intended to measure in a consistent and reliable manner. Further, exploratory factor analysis (EFA) was also conducted to describe the effects of the chosen constructs in the relationship.

4.4 Data analysis

After piloting the research instruments and finalizing the instruments, data from 430 contributors was collected and used to create a structural equation model (SEM), which was subsequently evaluated. The analysis for the current research utilized PLS measurement and structural models for validity and the relationship between the selected variables. In addition, Chat GPT was partly used for the improvement of manuscript writing particularly for the removal of grammatical errors. Following the recommendations of Jarvis et al. [27], all of the variables excluding mathematics beliefs were in the domain of first-order reflective type constructs. Only math beliefs were found in the domain of second-order constructs. The next phase involved identifying the nature of the constructs in the structural model. Based on this rule, the current study utilized a reflective-reflective type of model [3].

In current study, the measurement model with factor loading greater than 0.7 and Cronbach's alpha 0.9 had shown good reliability and convergent validity respectively. Table 1 displays overall results of the items. The acceptable values of factor loadings and AVE should be greater than or equal to 0.5. Similarly, CMR should be greater than or equal to 0.7 in order to demonstrate that the observed variables are internally consistent and reliable measures of the underlying factor.

Table 1: Detail of measurement model showing constructs and their subscales.

Construct	Subscale	Factor Loading	Cronbach's Alpha	CMR	AVE
Epistemological Beliefs	DP	0.83	0.90	0.93	0.72
	ST	0.85			
	UN	0.86			
	WP	0.86			
	EF	0.82			
Usefulness	UF1	0.90	0.93	0.95	0.77
	UF2	0.89			
	UF3	0.90			
	UF4	0.86			
	UF5	0.87			
	UF6	0.84			
Mastery Goal	MA1	0.89	0.93	0.95	0.74
	MA2	0.87			
	MA3	0.84			
	MA4	0.87			
	MA5	0.85			
	MA6	0.87			
Performance Goal	PER1	0.85	0.89	0.92	0.70
	PER2	0.85			
	PER3	0.85			
	PER4	0.87			
	PER5	0.75			

Construct	Subscale	Factor Loading	Cronbach’s Alpha	CMR	AVE
Avoidance Goal	AV1	0.88	0.95	0.96	0.82
	AV2	0.90			
	AV3	0.90			
	AV4	0.92			
	AV5	0.93			
	AV6	0.90			
Critical Thinking	CR1	0.77	0.81	0.87	0.58
	CR2	0.78			
	CR3	0.75			
	CR4	0.74			
	CR5	0.77			
Elaboration	EL1	0.80	0.91	0.93	0.69
	EL2	0.84			
	EL3	0.84			
	EL4	0.85			
	EL5	0.82			
	EL6	0.83			

Note: DP = Duration of Problem; ST = Steps; UN = Understandings; WP = Word Problems; EF = Effort; UF = Usefulness; MA = Mastery; PER = Performance; AV = Avoidance; CR = Critical Thinking; EL = Elaboration.

5 Results

Table 2 shows that every component in the model met the prerequisites for discriminant validity, which indicates the constructs are distinct and assess diverse core constructs. Additionally, it is noted that the item loadings of each construct were highest on their respective connected constructs, which further ensured the convergent validity of the model [20]. To assess the model predictive accuracy and significance R^2 , Q^2 and q^2 were calculated. Following Chin et al. [10], R^2 value of 0.67, 0.33 and 0.19 signify substantial, moderate, and weak predictive accuracy, respectively. The current model’s R^2 of 0.64 indicates a moderate effect size for DE problem solving [21]. 64% of the variability in solving differential equation can be linked to the factors of epistemological math beliefs, usefulness, performance, mastery, avoidance goal, critical thinking, and elaboration.

Table 2: Discriminant validity using Fornell–Larcker criterion.

Constructs	AV	CR	DEPS	EL	EMB	MA	PR	UF
AV	0.91							
CR	0.03	0.76						
DEPS	-0.26	0.33	Single item					
EL	0.04	0.44	0.61	0.83				
EMB	-0.01	0.40	0.65	0.72	0.85			
MA	-0.01	0.33	0.68	0.68	0.73	0.86		
PR	0.07	0.37	0.53	0.61	0.61	0.64	0.83	
UF	-0.08	0.28	0.66	0.52	0.62	0.67	0.42	0.88

Note: AV: Avoidance, CR: Critical thinking, DEPS: DE problem solving, EL: Elaboration, MA: Mastery, PR: Performance, EMB: Epistemological math beliefs, UF: Usefulness.

Additionally, epistemological and usefulness beliefs had strong effects on mastery goal (explaining 61% of the variance) and moderate effects on performance goal (explaining 38% of the variance). However, the avoidance goal had only a small effect. Finally, the study found that critical thinking and elaboration had weak and moderate effects on the DE task-solving construct, with R^2 values of 0.59 and 0.19, respectively (see Table 3). Table 3 shows a Goodness-of-Fit (GoF) value of 0.55. Using established cutoff values ($GoF_{small}=0.1$, $GoF_{medium}=0.25$, $GoF_{large}=0.36$), the model demonstrates a good fit to the current data [25]. Furthermore, the large GoF value indicates a substantial effect size. Similarly, the f^2 values, categorized according to Cohen [12] criteria (0.02, 0.15, and 0.35 for small, medium, and large, respectively), also suggest a substantial effect, respectively.

Table 3: Detail of R^2 and GoF estimations.

Selected Constructs	AVE	R^2	Effect Size	GoF ($\sqrt{AVE \times R^2}$)
AV	0.82	0.009	Small	0.55
CR	0.58	0.188	Small	
DEPS	1.00	0.64	Substantial	
EL	0.69	0.591	Medium	
MA	0.74	0.611	Medium	
PR	0.70	0.377	Medium	
UF	0.77	–		
EMB	0.72			

Note: AV: Avoidance, CR: Critical Thinking, DEPS: DE Problem Solving, EL: Elaboration, MA: Mastery, PR: Performance, EMB: Epistemological Math Beliefs, UF: Usefulness.

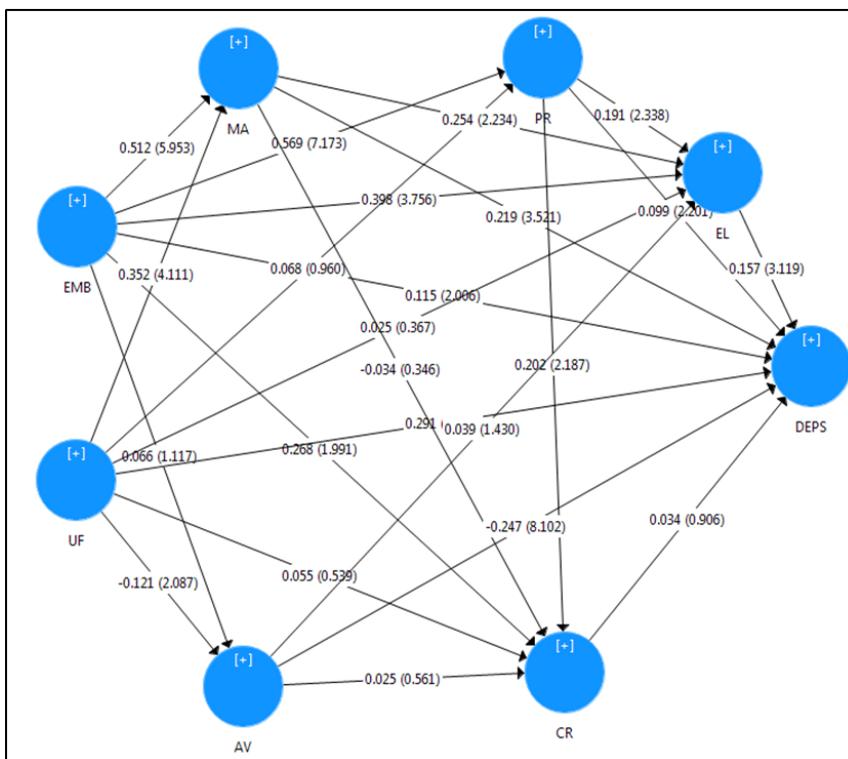


Figure 2: Complete structural model for established for the current study.

Based on the f^2 values (see Table 4), it was found that the avoidance goal (0.167) had a medium effect. Similarly, it was observed that mastery and performance goals have effectively mediated between the beliefs and problem solving with f^2 values of 0.412 and 0.318. However, the effect of avoidance goal is lesser (0.03). Interestingly, usefulness had better v values (0.195) for mastery goals only (see Table 5). Using blindfolding techniques and cross-validated redundancy, Q^2 was evaluated in the current study (Hair et al. [22]). All components' Q^2 values were greater than zero, as seen in Table 5, suggesting significant predictive relevance [20].

Table 4: Q^2 and f^2 values for the established model.

Constructs	Q^2	f^2 (MA)	f^2 (PR)	f^2 (AV)	f^2 (CR)	f^2 (EL)	f^2 (DEPS)
AV	0.007	–	–	–	0.001	0.004	0.167
CR	0.094	–	–	–	–	–	0.002
DEPS	0.620	–	–	–	–	–	–
EL	0.401	–	–	–	–	–	0.027
MA	0.448	–	–	–	0.000	0.053	0.043
PR	0.259	–	–	–	0.027	0.048	0.020
EMB	–	0.412*	0.318*	0.003	0.035	0.154	0.020
UF	–	0.195	0.002	0.005	0.002	0.001	0.118

Note: AV: Avoidance, CR: Critical Thinking, DEPS: DE Problem Solving, EL: Elaboration, MA: Mastery, PR: Performance, EMB: Epistemological Math Beliefs, UF: Usefulness. *Significant values.

5.1 Evaluation of structural model

A structural model was assessed in the second step in order to investigate the proposed correlation within the model. There are structural estimations in Table 6. Figure 2 shows the entire model with both direct and indirect effects in a manner that is easy to comprehend. The avoidance aim impacted negatively, whereas every other goal had a positive effect. In order to measure the mediations, three distinct paths were examined. The following sections contain the details.

Table 5: Detail of direct paths analysis (evaluation of Objective 1).

Selected Path	β values	SE values	T values	p values
EMB → DEPS	0.124	0.057	2.100	0.010
UF → DEPS	0.290	0.072	4.050	0.000
MA → DEPS	0.220	0.062	3.520	0.000
PR → DEPS	0.100	0.045	2.200	0.010
AV → DEPS	-0.250	0.031	8.100	0.000
CR → DEPS	0.035	0.037	0.910	0.180
EL → DEPS	0.160	0.050	3.120	0.000

Note: AV = Avoidance; CR = Critical Thinking; DEPS = DE Problem Solving; EL = Elaboration; MA = Mastery; PR = Performance; EMB = Epistemological Math Beliefs; UF = Usefulness.

Table 6: Analysis for mediating role of SRL strategies between goal orientations and DE problem solving.

Mediation	β	SE	T Value	p value	Findings	Domain
MA → EL → DEPS	0.039	0.020	1.82	0.03	P.M	Objective 4
PR → EL → DEPS	0.030	0.010	1.96	0.02	P.M	
AV → EL → DEPS	0.006	0.004	1.32	0.09	N.M	
MA → CR → DEPS	-0.001	0.003	-0.32	0.37	N.M	
PR → CR → DEPS	0.007	0.008	0.85	0.19	N.M	
AV → CR → DEPS	0.008	0.001	0.48	0.31	N.M	

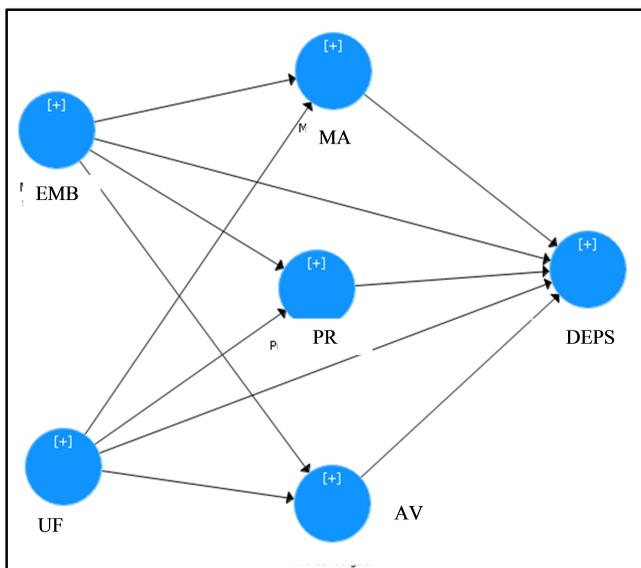


Figure 3: Goal orientations mediating among beliefs and DE problem solving.

5.1.1 The relation between student’s epistemological beliefs and problem solving via the mediatory variables of goal orientations

Using the structural estimates of Figure 3, effect of student’s beliefs through “students’ goal” was assessed. The results had shown partial mediation via performance and mastery goals (Table 6). Though, avoidance goal had shown adverse effects for solving the DE tasks. Potential reasons have been elaborated and discussed in discussion section.

5.1.2 Analyzing the relation between student’s epistemological beliefs and problem solving via the mediatory variables of self-regulated learning strategies

The structural estimates of the chosen path were taken into consideration in order to investigate the impact of students’ epistemological beliefs through the mediatory variable "SRL" (Figure 4). Table 7 indicates that the only one construct “elaboration” had mediated to solve the DE problems was elaboration. Interestingly, other constructs even critical thinking had not mediated. This might be occurred due to few notable reasons, which are elaborated in discussion section.

Table 7: Analysis for mediating role of SRL strategies between epistemological math beliefs and DE problem solving.

Mediation	β	SE	T Value	p value	Findings	Domain
EMB → EL → DEPS	0.062	0.025	2.40	0.000	P.M	Objective 2
EMB → CR → DEPS	0.009	0.010	0.830	0.200	N.M	–
UF → EL → DEPS	0.003	0.010	0.350	0.360	N.M	Objective 3
UF → CR → DEPS	0.001	0.004	0.460	0.320	N.M	–

Note: P.M = Partial Mediation; N.M = No Mediation; CR = Critical Thinking; EL = Elaboration; DEPS = DE Problem Solving; EMB = Epistemological Math Beliefs; UF = Usefulness.

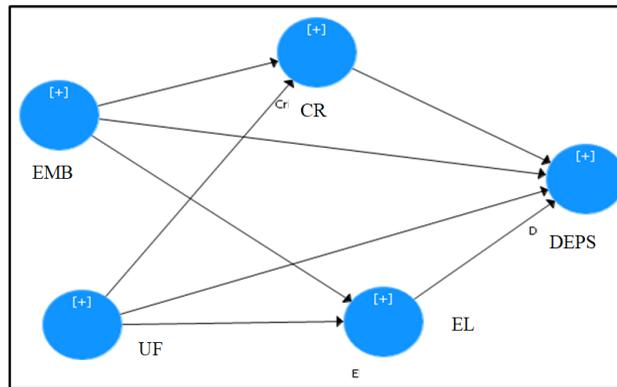


Figure 4: Mediating role of critical thinking and elaboration among beliefs and problem solving.

5.1.3 The relationship between student’s goals and problem solving via the mediatory variables of self-regulated learning strategies

The impact of students’ goals through the mediating variable "SRL" (Self-Regulated Learning) was evaluated using the structural estimations shown in Figure 5. Among the different goal orientations analyzed, only the elaboration showed a partial mediating effect in the entire process of solving Differential Equations (DE) problems, as shown in Table 8.

Table 8: Analysis for mediating role of SRL strategies between goal orientations and DE problem solving.

Mediation	β	SE	T Value	p value	Findings	Domain
MA → EL → DEPS	0.039	0.020	1.82	0.03	P.M	Objective 4
PR → EL → DEPS	0.030	0.010	1.96	0.02	P.M	
AV → EL → DEPS	0.006	0.004	1.32	0.09	N.M	
MA → CR → DEPS	-0.001	0.003	-0.32	0.37	N.M	
PR → CR → DEPS	0.007	0.008	0.85	0.19	N.M	
AV → CR → DEPS	0.008	0.001	0.48	0.31	N.M	

Note: PM = Partial Mediation; NM = No Mediation; EL = Elaboration; CR = Critical Thinking; MA = Mastery; PR = Performance; AV = Avoidance; DEPS = DE Problem Solving.

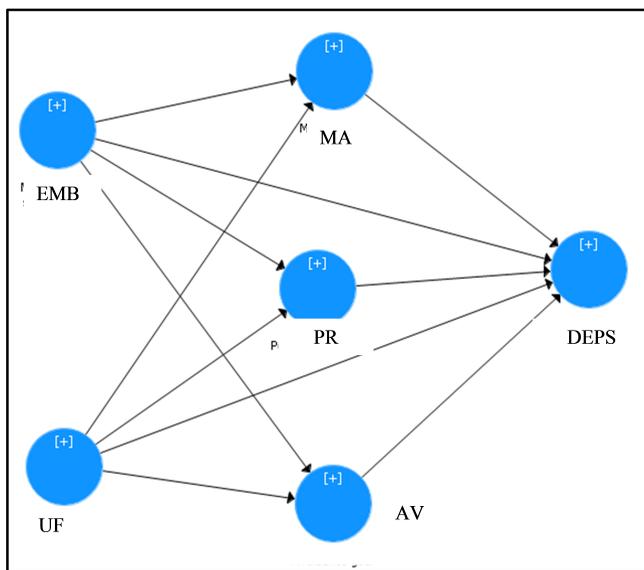


Figure 5: Critical thinking and elaboration mediating among goal orientations and problem solving.

6 Discussion

The results show that self-regulated learning (particularly elaboration strategies), perceived usefulness, epistemological math beliefs, and goal orientations (both mastery and performance) are all strongly correlated. Critical thinking and avoidance behaviors, on the other hand, did not significantly affect the analysis. Mastery and performance goals were found to have a strong mediating influence between beliefs and problem solving while considering the mediating effect. The relationship between beliefs and problem-solving has also been demonstrated via elaborations. Ultimately, elaboration also acted as a mediator between mastery and performance goal.

6.1 Examining direct effects of beliefs on problem solving

Findings from the SEM analysis showed that problem-solving skills were strongly impacted by epistemological beliefs about mathematics. Interestingly, the most positive influence was found in useful beliefs. This suggests that students who perceived the value of mathematics tended to be more deeply engaged cognitively and had improved problem-solving abilities [46]. Surprisingly when comparing data, respondents from the public sector put greater emphasis on usefulness of mathematics and epistemological beliefs pertaining to problem solving than did respondents from the private sector. For the teaching and learning of Differential Equations (DE) courses, the current study highlights an array significant finding. Students who believe they can't handle extensive problems, instance, certain times consider that most mathematics tasks ought to be completed promptly. When assigned more challenging assignments, these students frequently opt not to address the problems completely and proceed ahead to other ones [32]. This leads to fixed mind set [22]. Therefore, it is the responsibility of teachers to assist students determine a growth mindset and take a "can-do" attitude regarding DE assignments. The social norm, such as the marriage of young girls, could be another factor. From UN report [40], over 63% of girls from low-income families and even 24% of girls from high-income families get married before turning

18. In certain instances, women's job opportunities are limited and distinct from those of boys. The majority of women do not groom themselves properly and may not have the ability to make decisions because of a lack of resources and opportunities. Students' attitudes toward learning and problem-solving are significantly impacted by these factors [33].

6.2 Direct effects of goal orientation on problem solving

The mastery goal had shown a positive effect on DE problem-solving. Interestingly, mastery goal had a greater influence on DE problem-solving as compared to the performance goal. Though, the avoidance goal had negative effect. To determine the effect of goal orientation on students' problem-solving ability, results were also compared gender wise and educational sector wise. Possible reasons to illustrate these findings are provided in the following sections.

6.2.1 An explanation for mastery goal supporting the problem solving

In the current study, the role of mastery goal was significant. Internally inspired mastery-oriented students having focus on their tasks were successful in problem solving. Hence, researchers perceived that those participants considering learning for their self-sake and had emphasized on improving skills, were better in DE problem solving [39, 17]. Results from this study are in accordance with the findings of earlier research [6]. There are several factors that influence public sector participants' mastery goal orientation, such as the organizational environment, which may predict how teachers will instruct and how students will pursue their goals [13]. Since the government runs public schools, the availability of greater resources such as infrastructure, labs, and trained teachers may be the reason. Another potential reason could be the mainstream education sourcing as a result of which students begin to embrace their own goals and values, which focuses more on mastery orientation [6]. Private sector schools put a greater emphasis on students' grades and performance. Additionally, the availability of additional resources, a supportive environment, and assistance that encourage students to accomplish better are possible reasons for the male participants' tendency towards the mastery goal orientation. In developing countries like Pakistan, Grocery shopping, rent, utilities, medical care, and other food expenses are all the responsibility of man. As a result, majority of families find out funding for boys' education as an investment in their future, but girls do not receive the same level of educational assistance. Men's earnings become relatively high as a result of holding superior social positions, and they are also perceived as more capable than women, making gender an important yardstick of reputation in society.

6.2.2 Clarification for performance goal starring the problem solving

The role of performance goal was also significant in the current research. The results of the data comparison for performance goals were consistent for both sectors. Moreover, female participants also shown higher importance for the performance goal. Outperforming others was the primary goal for performance-oriented students, which typically encourages rote memorization and less attention to learning something novel and challenging. Post-secondary education is a critical period in students' lives in developing nations, and it puts constant pressure on educators and learners to perform well in order to gain admission to professional universities and employment. This encourages test-taking and superficial memorizing skills. Teachers' expectations of their students influence their mindsets and behaviors. Similarly, parental expectations and perspectives

are a fundamental factor of an individual's performance [36]. Students develop educational goals based on their understanding of parental nurturing, which has an effect on their academic performance [9]. Basit and Rahman [2] in their study in Pakistani context also justified the findings of current study. Therefore, parents of targeted population for this study also had expectation from their children to outperform others to meet their opportunities.

6.2.3 Possible reasons for avoidance goal not supporting the problem solving

The avoidance goal had an adverse effect in the current research. Possible reason for this might be the high expectation of parents and educator to outperform others had promoted avoidance goal in students [29]. By comparing data, it was revealed that respondents from the private sector were more likely to have avoidance goal. Additionally, in terms of the avoidance goal, both genders were on equal level [1]. Students who admit an avoidance goal are more likely to display defensive and impassive behaviors, leading to a withdrawal from learning and hindering self-regulation. These people have distinctive patterns of neural activity that influence how they allocate cognitive effort and perceive values, which further reinforces their avoidance behaviors [31]. Avoidance goals—those focused on avoiding failure, errors, or negative judgments—do not support effective math problem solving because they can increase anxiety, reduce motivation, and lead to withdrawal from challenging tasks. When students are primarily focused on avoiding errors, they may become risk-averse, less persistent when faced with difficult problems, and less likely to engage in exploratory or strategic thinking.

This mindset hampers the development of problem-solving skills, as students may prioritize avoiding failure over understanding concepts or applying strategies. Research suggests that goal orientations emphasizing mastery and learning are more conducive to successful problem solving. In contrast, avoidance goals tend to undermine intrinsic motivation and hinder persistence, which are critical for mastering complex mathematical tasks [31]. The assessment system emphasizes on how much bookish knowledge students have memorized rather than evaluating them based on newly learned skills, situational awareness, and coping with the topic's general idea. This system brings avoid performance goals in pupils. When data was compared, respondents from the private sector tended to be more performance avoidance oriented. Other researchers believed that educational culture that took place in private sectors, increases the expectations of parents, and instructors from the students for their accomplishment [6]. These high expectations and the likelihood of failure could make people feel more anxious [49]. These factors encouraged the performance avoidance goal [29].

6.3 Direct effects of SRL and potential reasons of critical thinking not supporting the problem solving

It was anticipated that employing SRL techniques (elaboration and critical thinking) would help students solve problems more effectively [43]. The findings showed that problem solving and elaboration had a high correlation, while critical thinking had no apparent effect. A possible reason could be that the current study only used a limited number of learning techniques (such as elaboration and critical thinking), which is why critical thinking did not have as much of an impact as it did in earlier research by Fadlelmula et al. [17]. Another reason could be the self-reporting traits of the data, which, Muis [35] shows, led to conceptual and technological problems. A classroom environment that actively engages students in solving real-world problems is essential for critical thinking. In particular, it calls for patience, time, practice, and training [43]. These results

can be fitted to the assessment methods employed in Pakistani educational systems. Students may therefore be uninterested to endorse critical thinking in problem solving.

6.3.1 Partial mediation of goal orientations between the epistemological and useful beliefs and problem solving

The outcomes of SEM analysis confirmed that mastery and performance goals had mediated the epistemological beliefs on solving DE tasks. The indirect effect even after including goal orientations as a mediator was less significant as compared to direct effect. Therefore, the results had endorsed the commonly held belief that performance and mastery goals boost student's capability to solve problems. Nevertheless, contrary to common belief, these (performance and mastery goals) had partially mediated. These findings also suggest that there are other factors besides goal orientations that contribute to the enhancement of student's problem-solving skills through epistemological beliefs. Current results are consistent with previous studies that provide significant evidence for the mediation role of epistemological beliefs in predicting goal orientation. The current findings are in line with the prior researches having strongly support for the mediating role of epistemological beliefs in predicting goal orientations [24]. Similar positive relationship between epistemological beliefs and goal orientation was reported by Kizilgunes et al. [30]. Overall, it can be stated that students who were motivated and had positive attitude towards mathematics tend to solve more DE problems.

The current findings are inconsistent with previous research when it pertains to usefulness beliefs. Additionally, it can be concluded that usefulness beliefs might influence factors besides goal orientations [48]. Schommer Aikins and Duell [46] further research also revealed an indirect effect of believing in the usefulness of mathematics. Overall, it can be claimed that students' cognitive depth and problem-solving abilities improve with their belief in the value of mathematics. Other factors may include the side wise effect of SRL strategies that may be led to change in the students' engagement. Therefore, mediation analysis of SRL strategies was also carried out separately in the following sections.

6.3.2 SRL showing partial mediation of between beliefs and problem solving

Likewise, it was anticipated that goal orientation and SRL techniques would strengthen students' useful and epistemological beliefs, which would in turn enhance their problem-solving skills. However, only elaboration showed partial mediation for solving differential equation (DE) problems, which was unexpected. It can be concluded that the enhancement in problem-solving resulting from epistemological and usefulness beliefs is probably due to other reasons besides SRL strategies. The results of this study indicated that elaboration may enhance the relationship between solving DE tasks and epistemological math beliefs, which is consistent with previous studies [26]. Furthermore, Was [52] showed a relationship between students' problem-solving skills and a variety of indicators of their SRL approaches. Author argued that the association between solving DE tasks and epistemological math beliefs can be enhanced via elaboration. The current study demonstrated that students' ability to employ critical thinking to solve DE tasks was not influenced by their epistemological math beliefs, which is contrary to assumptions and certain prior research. These results partially contradict Schommer [45], hypothesis that SRL strategies, and epistemological convictions are strongly correlated to students' achievements. According to Muis [35], conceptual and technological problems with self-reported measures may be the cause of the observed discrepancies. Muis pointed out that these conceptual and methodological limitations hampered the potential contribution of studies that connected epistemological views to

academic, cognitive, and motivational results by relying solely on self-report.

Consistent with previous research, the current findings indicate that elaboration and critical thinking have no mediating impact on usefulness beliefs [48] (such as performance in solving problems). On the other hand, Schommer Aikins and Duell [46] found an indirect impact from belief in the usefulness of mathematics. This indicates that it has no direct impact on problem solving. A possible explanation for this contradiction could be the lack of relevant research studies and the challenge of assessing accurately usefulness beliefs during DE tasks. Research on the mediating role of self-regulated learning (SRL) between the ability to solve differential equation (DE)-based problems and usefulness beliefs was limited. As a result, our study provides unique perspectives on SRL's mediating role in this relationship. Mediation analysis also indicated the side wise effect of SRL strategies due to the correlation between the students' goal orientations and solving DE based tasks. This relationship that may be led to change in the students' extra engagement. Therefore, mediation analysis of SRL strategies carried out separately. A detailed explanation is provided in the following sections.

6.3.3 Partial mediatory role of SRL strategies among student's goals and problem solving

This study concluded that elaboration can mediate among mastery and performance goals on DE problem solving. Possible reason for this mediation might be the requirements of the high grades, which had led to the utilization of more strategies to achieve prominent results. current outcomes are well aligned with Kadioglu and Uzuntiryaki Kondakci [29]. As a whole, the performance goal's results were more encouraging and noteworthy. These findings can be clarified the pervasive assessments and teaching practices in developing countries like Pakistan, where grade-focused evaluation dominates. As a result, the performance goal's role in solving DE based tasks appeared plausible.

7 Implications

Current study has led to significant findings for the teaching of differential equations (DE). It emphasizes how students may anticipate most mathematical tasks to be completed rapidly if they believe they are incapable of solving time-consuming problems. As a result, these students might ignore increasingly difficult coursework. The study highlights how essential it is for teachers to inspire students and foster a "can-do" mindset in resolving problems. For efficient mathematical problem solving, we consequently encourage educators and policymakers to integrate these aspects.

Few studies have been conducted on the usefulness beliefs of DE-based problem-solving. This study demonstrates that the belief that mathematics is useful is a strong cognitive motivator that proactively enhances problem-solving skills and encourages a deeper understanding of the topic. Therefore, one of the main goals of mathematics education should be to foster these positive usefulness beliefs. This study's key contribution lies in demonstrating the mediating role of self-regulated learning (SRL) among beliefs in usefulness of mathematics and solving DE based course was not studied prior to present research.

The present work also indicates that in the selected population, role of performance goal was prominent, whereas the role of critical thinking was insignificant. In developing nations like Pak-

istan, where both educators and parents desire their children to perform better than others so as to fulfill the expectations they have, these results can be customized to the educational systems of those nations. Such an educational system encourages rote memorization rather than evaluating students based on their ability to learn new skills, comprehend situations, and cope with the topic's fundamental notion. For this reason, teachers should be trained for establishing educational environments that proactively supports students' performance and mastery goals through appropriate SRL-strategies including critical thinking to enhance students' problem-solving skills. Additionally, through seminars, television, and social media, an initiative should be started to raise awareness among parents, educators, and wider society about the negative effects of parents' unrealistic goals and superficial memorization for achieving excellent grades. It is essential to foster a culture that supports comprehensive, deliberate, and successful teaching and learning.

8 Conclusions

Current study showed a substantial correlation between goal orientation (mastery and performance), SRL (elaboration only), perceived usefulness of the subject, and epistemological math beliefs for calculus-based problem-solving. Avoidance and critical thinking were found to have no apparent effect. The mediating effect was examined, and it revealed that beliefs and problem solving were mediated by mastery and performance goals. Likewise, elaboration have demonstrated the powerful mediating role between beliefs and problem-solving. Elaboration ultimately acted as a mediator between mastery and performance goal as well. These results show how these factors may be employed to help students become better DE problem solvers and make DE learning more effective and impactful.

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